SC4012
WASSCE 2019
FURTHER MATHEMATICS/
MATHEMATICS (ELECTIVE)
2½ hours

2

Name.....

Index Number.....

THE WEST AFRICAN EXAMINATIONS COUNCIL

West African Senior School Certificate Examination
for School Candidates

SC 2019 FURTHER MATHEMATICS/MATHEMATICS (ELECTIVE) 2 2½ hours
[100 marks]

Write your **name** *and* **index number** *in* **ink** *in the spaces provided above.*

Answer **twelve** *questions in all.* **All** *the* **eight** *questions in Section* A *and* **four** *questions from Section* B *with* **at least one** *from* **each** *part.*

In each question, all necessary details of working, including rough work, **must** *be shown with the answer.*

Give answers as accurately as data and tables allow.

Graph papers are provided for your use in the examination.

The use of non-programmable, silent and cordless calculator is allowed.

SECTION A
[48 marks]

Answer **all** *the questions in this section. All questions carry equal marks.*

1. Two independent events K and L are such that p(K) = x, p(L) = $(x + \frac{1}{5})$ and p(K ∩ L) = $\frac{3}{20}$. Find the value of x.

2. Seven participants in an art contest were ranked by two judges as follows:

Participant	**A**	**B**	**C**	**D**	**E**	**F**	**G**
1st Judge	3	4	1	6	5	7	2
2nd Judge	3	6	2	5	7	4	1

(*a*) Calculate, correct to **three** decimal places, the Spearman's rank correlation coefficient for the scores of the judges.

(*b*) Comment on your results.

3. $\mathbf{F}_1$ (3 N, 030°), $\mathbf{F}_2$ (4 N, 090°), $\mathbf{F}_3$ (6 N, 135°) and $\mathbf{F}_4$ (7 N, 240°) act on a particle. Find, correct to **two** decimal places, the magnitude of the resultant force.

4. A uniform pole, **PQ**, 30 m long and of mass 4 kg is carried by a boy at P and a man 8 m away from Q. Find the distance from P where a mass of 20 kg should be attached so that the man's support is twice that of the boy, if the system is in equilibrium.

[Take g = 10 ms^{-2}]

5. Solve: $3x^{\frac{1}{2}} + 5 - 2x^{-\frac{1}{2}} = 0$.

6. A point **P** divides the straight line joining **X** (1, –2) and **Y** (5, 3) internally in a ratio 2 : 3. Find the:
(*a*) coordinates of **P**;

(*b*) equation of the straight line that passes through **N** (3, –5) and **P**.

7. (*a*) Find the sum of the series: 32 + 8 + 2 + ...

(*b*) Simplify: $\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}} - \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}$.

8. Without using Mathematical tables or calculator, find, in surd form (radicals), the value of tan 22.5°.

SECTION B
[52 marks]

Answer **four** *questions* **only** *from this section with* **at least one** *question from* **each** *part.*

All *questions carry* **equal** *marks.*

PART I
PURE MATHEMATICS

9. (*a*) Find the range of values of x for which $2x^2 \geq 9x + 5$.

(*b*) (i) Write down in ascending powers of x the binomial expansion of $(2 + x)^6 - (2 - x)^6$.

(ii) Using the result in (*b*)(i), evaluate $(2.01)^6 - (1.99)^6$, correct to **four** decimal places.

10. A circle $x^2 + y^2 - 2x - 4y - 5 = 0$ with centre O is cut by a line $y = 2x + 5$ at points P and Q. Show that $\overline{QO}$ is perpendicular to $\overline{PO}$.

11. (*a*) Given that $M = \begin{pmatrix} 3 & -5 \\ 4 & 2 \end{pmatrix}$, find:

(i) M^{-1}, inverse of M.

(ii) the image of $(1, -1)$ under M^{-1}.

(*b*) Two linear transformations P and Q, are defined by $P: (x, y) \rightarrow (5x + 3y, 6x + 4y)$ and $Q: (x, y) \rightarrow (4x - 3y, -6x + 5y)$.

(i) Write down the matrices P and Q.

(ii) Find the matrix R defined by $R = PQ$.

(iii) Deduce Q^{-1}, the inverse of Q.

PART II
STATISTICS AND PROBABILITY

12. A box contains 5 blue, 7 green and 4 red identical balls. **Three** balls are picked from the box one after the other without replacement. Find, the probability of picking:

(*a*) **two** green balls and **a** blue ball;

(*b*) **no** blue ball;

(*c*) **at least** one green ball;

(*d*) **three** balls of the same colour.

13. The ages, x (in years), of a group of 18 adults have the following statistics:
$\Sigma x = 745$ and $\Sigma x^2 = 33951$.

(*a*) Calculate the:

(i) mean age;

(ii) standard deviation of the ages of the adults, correct to **two** decimal places.

(*b*) One person leaves the group and the mean age of the remaining 17 is 41 years. Find the:

(i) age of the person who left;

(ii) standard deviation of the remaining 17 adults, correct to **two** decimal places.

PART III

VECTORS AND MECHANICS

14. Three forces $0\mathbf{i} - 63\mathbf{j}$, $32.14\mathbf{i} + 38.3\mathbf{j}$ and $14\mathbf{i} - 24.25\mathbf{j}$ act on a body of mass 5 kg. Find, correct to the **nearest** whole number, the:

(*a*) magnitude of the resultant force;

(*b*) direction of the resultant force;

(*c*) acceleration of the body.

15. Two particles **P** and **Q** move towards each other along a straight line *MN*, 51 metres long. **P** starts from *M* with velocity 5 m s^{-1} and constant acceleration of 1 m s^{-2}. **Q** starts from *N* at the same time with velocity 6 m s^{-1} and at a constant acceleration of 3 m s^{-2}. Find the time when the:

(*a*) particles are 30 metres apart;

(*b*) particles meet;

(*c*) velocity of **P** is $\frac{3}{4}$ of the velocity of **Q**.

END OF PAPER

SC4011
WASSCE 2019
FURTHER MATHEMATICS/
MATHEMATICS (ELECTIVE)1
Objective Test
1½ hours

1

Name.....

Index Number.....

THE WEST AFRICAN EXAMINATIONS COUNCIL

West African Senior School Certificate Examination for School Candidates

SC 2019 FURTHER MATHEMATICS/ MATHEMATICS (ELECTIVE) 1 1½ hours

OBJECTIVE TEST

[40 marks]

Do **not** *open this booklet until you are told to do so. While you are waiting, read and observe the following instructions carefully. Write your* **name** *and* **index number** *in the spaces provided above.*

Answer **all** *the questions on your Objective Test answer sheet.*

1. Use 2B pencil throughout.
2. On the pre-printed answer sheet, check that the following details are **correctly** printed:
   (*a*) In the space marked *Name*, check your **surname** followed by your **other names**.
   (*b*) In the spaces marked *Examination*,*Year, Subject* and *Paper*, check 'WASSCE', 'SC 2019', 'FURTHER MATHEMATICS/ MATHEMATICS (ELECTIVE)', and '1' in that order.
   (*c*) In the box marked *Index Number,* your **index number** has been printed vertically in the spaces on the left-hand side, and each numbered space has been shaded in line with each digit. **Reshade** each of the shaded spaces.
   (*d*) In the box marked *Subject Code*, the digits 401112 are printed vertically in the spaces on the left-hand side. **Reshade** the corresponding numbered spaces as you did for your index number.
3. An example is given below. This is for a female candidate whose *name* is Mariam Esi KWAO. Her *index number* is 7102143958 and she is offering *Further Mathematics/Mathematics(Elective)* 1.

THE WEST AFRICAN EXAMINATIONS COUNCIL

ANSWER SHEET

PRINTED IN BLOCK LETTERS.
Name: KWAO MARIAM ESI | GHA
Examination: WASSCE | Year: SC 2019
Subject: FURTHER MATHS. / MATHS. (ELECTIVE) | Paper: 1

INSTRUCTIONS TO CANDIDATES
1. Use grade 2B pencil throughout.
2. Answer each question by choosing one letter and shading it like this: ⊏A⊐ ■ ⊏C⊐ ⊏D⊐ ⊏E⊐
3. Erase completely any answer you wish to change.
4. Leave extra spaces blank if the answer spaces provided are more than you need.
5. Do not make any markings across the heavy black marks at the right hand edge of your answer sheet.

INDEX NUMBER	0	1	2	3	4	5	6	7	8	9
7									■	
1		■								
0	■									
2			■							
1		■								
4					■					
3				■						
9										■
5						■				
8							■			

SUBJECT CODE	0	1	2	3	4	5	6	7	8	9
4					■					
0	■									
1		■								
1		■								
1		■								
2			■							

For Supervisors only
If candidate is absent shade this space. ▭

Answer **all** *the questions.*

Each question is followed by four options lettered A *to* D. *Find the* **correct** *option for* **each** *question and shade in* **pencil** *on your answer sheet, the answer space which bears the same letter as the option you have chosen. Give only* **one** *answer to* **each** *question. An example is given below.*

The ages, in years, of four boys are 10, 12, 14 and 16. What is the mean age of the boys?

A. 12 years
B. $12\frac{1}{2}$ years
C. 13 years
D. $13\frac{1}{2}$ years

The correct answer is 13 years, *which is lettered* C, *and therefore answer space* C *would be shaded:* ⊏A⊐ ⊏B⊐ ⊏C⊐ ⊏D⊐ ⊏E⊐

Think carefully before you shade the answer spaces; erase completely any answers you wish to change.

Do all rough work on this question paper.

Now answer the following questions:

1. Solve: $8^{x-2} = 4^{3x}$.

A. –1
B. –2
C. 1
D. 2

2. Evaluate: $\tan 75^{\circ}$, leaving the answer in surd form (radicals).

A. $\sqrt{3} + 1$
B. $\sqrt{3} - 1$
C. $\sqrt{3} + 2$
D. $\sqrt{3} - 2$

3. Solve $\frac{p}{2} + \frac{k}{3} = 5$ and $2p - k = 6$ simultaneously.

A. $p = 6, k = 6$
B. $p = -6, k = -6$
C. $p = 6, k = -6$
D. $p = -6, k = 6$

4. Rationalize: $\frac{1}{\sqrt{2} + 1}$.

A. $1 - \sqrt{2}$
B. $\frac{1 - \sqrt{2}}{2}$
C. $\sqrt{2} - 1$
D. $\frac{\sqrt{2} - 1}{2}$

5. If ${}^nC_2 = 15$, find the value of n.

A. 6
B. 5
C. 8
D. 7

6. An operation (*) is defined on the set $\mathbf{T} = \{-1, 0, ..., 5\}$ by x (*) $y = x + y - xy$.

Which of the following operation(s) will give an image which is an element of T?

I. 2 (*) 5
II. 3 (*) 2
III. 3 (*) 4

A. I and III only
B. II and III only
C. I only
D. II only

7. Given that $g : x \to 3x$ and $f : x \to \cos x$, find the value of $g \circ f(20^\circ)$.

A. 0.94
B. 0.50
C. 2.82
D. 2.60

8. A linear transformation is defined by $\mathbf{T} : (x, y) \to (-x + y, -4y)$. Find the image, Q′, of Q(−3, 2) under **T**.

A. Q′(5, −3)
B. Q′(−5, −8)
C. Q′(5, −8)
D. Q′(−8, 5)

9. If $g : r \to 5 - 2r$, r is a real number, find the image of −3.

A. 11
B. −9
C. 13
D. −1

10. Consider the following statements:
p : Birds fly
q : The sky is blue
r : The grass is green

What is the symbolic representation of "If the grass is green and the sky is not blue, then the birds do not fly"?

A. $(r \wedge \sim q) \Rightarrow \sim p$
B. $(r \wedge p) \Rightarrow q$
C. $(r \wedge \sim p) \Rightarrow \sim q$
D. $(r \wedge q) \Rightarrow \sim p$

11. Given that $\frac{1}{x^2-4} \equiv \frac{P}{x+2} + \frac{Q}{x-2}$, $x \neq \pm 2$, find the value of $(P + Q)$.

A. $\frac{1}{2}$

B. 0

C. $\frac{3}{2}$

D. 1

12. Find the sum of the **first** 20 terms of the sequence: $-7, -3, 1, \ldots$.

A. 690

B. 620

C. 1240

D. 660

13. Find the value of x for which $6(\sqrt{4x^2+1}) = 13x$, where $x > 0$.

A. $\frac{24}{25}$

B. $\frac{5}{6}$

C. $\frac{6}{5}$

D. $\frac{25}{24}$

14. Calculate the distance between points $(-2, -5)$ and $(-1, 3)$.

A. $\sqrt{17}$ units

B. $\sqrt{5}$ units

C. $\sqrt{73}$ units

D. $\sqrt{65}$ units

15. If $P = \begin{pmatrix} 2 & 3 \\ -4 & 1 \end{pmatrix}$, $Q = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$ and $PQ = k\begin{pmatrix} 45 \\ -20 \end{pmatrix}$, find the value of k.

A. $\frac{4}{5}$

B. $-\frac{5}{4}$

C. $\frac{5}{4}$

D. $-\frac{4}{5}$

16. The second and fourth terms of an exponential sequence (G.P.) are $\frac{2}{9}$ and $\frac{8}{81}$ respectively. Find the sixth term of the sequence.

A. $\frac{1}{4}$

B. $\frac{32}{729}$

C. $\frac{81}{32}$

D. $\frac{9}{8}$

17. Points **X** and **Y** are on the same horizontal base as the foot of a building such that **X** is 96 m due east of the building and **Y** is due west. If the angle of elevation of the top of the building from **X** is 30° and that of **Y** is 60°, calculate the distance of **Y** from the building.

A. 32 m
B. 42 m
C. 50 m
D. 30 m

18. Find the coordinates of the point on the curve $y = 3x^2 - 2x - 5$, where the tangent is parallel to the line $y - 5 = 8x$.

A. $(0, \frac{5}{3})$

B. $(\frac{5}{3}, 0)$

C. $(-\frac{5}{3}, 0)$

D. $(0, -\frac{5}{3})$

19. If the mean of 2, 5, $(x + 1)$, $(x + 2)$, 7 and 9 is 6, find the median.

A. 5.5
B. 6.5
C. 5.0
D. 6.0

20. Calculate the mean deviation of 5, 8, 2, 9 and 6.

A. 4
B. 2
C. 5
D. 3

A particle starts from rest and moves in a straight line such that its velocity v m s^{-1} at time t seconds is given by $v = 3t^2 - 6t$.

Use the information to answer questions **21** *and* **22**.

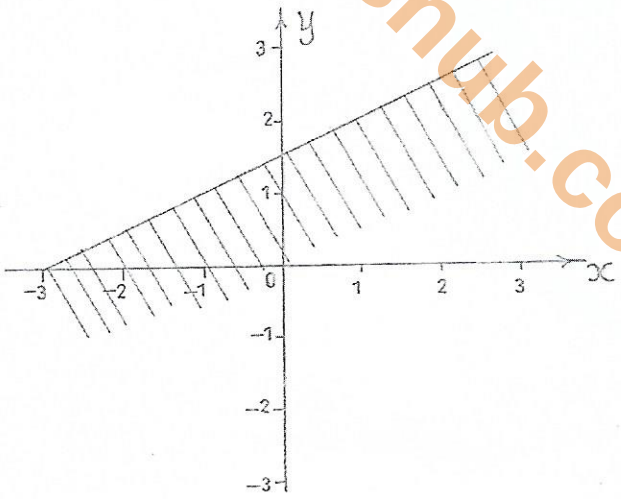
21. Calculate the distance in 4 seconds.

A. 16 m
B. 96 m
C. 12 m
D. 64 m

22. Calculate the acceleration in the 3rd second.
A. $3\ m\ s^{-2}$
B. $9\ m\ s^{-2}$
C. $0\ m\ s^{-2}$
D. $6\ m\ s^{-2}$

23. Find the constant term in the binomial expansion of $\left(2x^2+\frac{1}{x^2}\right)^4$.
A. 12
B. 42
C. 10
D. 24

24.



Which of these inequalities is represented by the shaded portion of the graph?
A. $2y - x + 3 < 0$
B. $2y + x + 3 < 0$
C. $2y + x - 3 < 0$
D. $2y - x - 3 < 0$

25. A 35 N force acts on a body of mass 5 kg for 2 seconds. Calculate the change in momentum of the body.
A. $50\ kg\ m\ s^{-1}$
B. $70\ kg\ m\ s^{-1}$
C. $35\ kg\ m\ s^{-1}$
D. $55\ kg\ m\ s^{-1}$

26. Solve, correct to **three** significant figures, $(0.3)^x = (0.5)^8$.
A. 0.461
B. 4.61
C. 0.0130
D. 4.606

27. Given that P and Q are two non-empty subsets of the universal set μ, find $P \cap (Q \cup Q')$.
A. P'
B. Q'
C. P
D. Q

28. Find the coefficient of the third term in the binomial expansion of $\left(2x+\frac{3y}{4}\right)^3$ in descending powers of x.
A. $\frac{27}{8}y^2$
B. $9y^2$
C. $\frac{27}{64}y^2$
D. $8y^2$

29. Find the coordinates of the centre of the circle $3x^2 + 3y^2 - 6x + 9y - 5 = 0$.

A. $(1, -\frac{3}{2})$

B. $(3, -\frac{9}{2})$

C. $(-3, \frac{9}{2})$

D. $(-1, \frac{3}{2})$

30. Evaluate: $\int_0^9 \sqrt{x}\, dx$.

A. 9
B. 3
C. 27
D. 18

31. The function $f: x \to x^2 + px + q$ has a turning point when $x = -3$ and a remainder of -6 when divided by $(x + 2)$. Find the value of q.

A. -2
B. 6
C. -8
D. 2

32. If $y = (5 - x)^{-3}$, find $\frac{dy}{dx}$.

A. $\frac{3}{(5-x)^4}$

B. $\frac{-15}{(5-x)^4}$

C. $\frac{15}{(5-x)^4}$

D. $\frac{-3}{(5-x)^4}$

33. Which of the following vectors is perpendicular to $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$?

A. $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

B. $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

C. $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

D. $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

34. Find, correct to the **nearest** degree, the angle between $\mathbf{p} = 12\mathbf{i} - 5\mathbf{j}$ and $\mathbf{q} = 4\mathbf{i} + 3\mathbf{j}$.
A. 75^o
B. 59^o
C. 76^o
D. 60^o

35. Find the area between line $y = x + 1$ and the x-axis from $x = -2$ to $x = 0$.
A. 2 square units
B. 1 square unit
C. 5 square units
D. 4 square units

36. How many numbers greater than 200 can be formed from the digits 1, 2, 3, 4, 5 if no digit is to be repeated in any particular number?
A. 288
B. 50
C. 300
D. 60

37. The probabilities that John and Jane will pass an examination are 0.9 and 0.7 respectively. Find the probability that **at least** one of them will pass the examination.
A. 0.72
B. 0.97
C. 0.28
D. 0.67

38. Given that X and Y are independent events such that $p(X) = 0.5$, $p(Y) = m$ and $p(X \cup Y) = 0.75$, find the value of m.
A. 0.4
B. 0.3
C. 0.6
D. 0.5

39. A uniform beam ,PQ, is 100 cm long and weighs 35 N. It is placed on a support at a point 40 cm from P. If weights of 54 N and **F** N are attached at P and Q respectively in order to keep it in a horizontal position, calculate, correct to the **nearest** whole number, the value of **F**.
A. 35
B. 30
C. 69
D. 60

40. Evaluate: $\lim_{x \to 1} \left(\frac{1 - x}{x^2 - 3x + 2} \right)$.
A. $\frac{1}{2}$
B. -1
C. 1
D. $-\frac{1}{2}$

END OF PAPER